ARITHMETIC AND GEOMETRY OF LOCAL / GLOBAL FIELDS TUAN CHAU, 25-29 JUNE 2018

Tomoyuki Abe (Kavli IPMU Tokyo, Japan) : Nearby cycles formalism in arithmetic $\mathcal{D}$-modules.

Abstract : I will explain how to construct the nearby cycle formalism in arithmetic $\mathcal{D}$-modules. As an application, I will discuss how to realize the category of overconvergent F-isocrystals purely in terms of arithmetic $\mathcal{D}$-module theory.

Keisuke Arai (Tokyo Denki University, Japan) : Points on Shimura curves rational over imaginary quadratic fields in the non-split case.

Abstract : For an imaginary quadratic field k of class number > 1, Jordan proved that there are only finitely many isomorphism classes of rational indefinite quaternion division algebras B such that the associated Shimura curve M^B has k-rational points and k splits B. We study the case where k does not split B, and solve the remaining part. In other words, the main result asserts that there are only finitely many isomorphism classes of B such that M^B has k-rational points.

Pascal Boyer (Université de Paris 13, France) : Automorphic congruences associated to torsion cohomology classes of some Shimura varieties.

Abstract : About the $\mathbb{Z}_l$-cohomology of Shimura varieties, on can be interested by killing the torsion using for example localization at some well chosen maximal ideal of some Hecke algebra. On the opposite direction, we can ask for the arithmetic meaning of torsion classes so that we are led to the problem of the construction of such classes. In this talk we will try to tackle this second point of view through the construction of automorphic congruences.

Florian Breuer (University of Newcastle, Australia) : Heights and Isogenies of Drinfeld modules.

Abstract : Consider two isogeneous $\mathbb{F}_q[t]$-Drinfeld modules of rank 2. We will show that the difference between the Weil heights of their j-invariants is bounded by a constant plus $(q^2 - 1)$ times the logarithm of the degree of the isogeny. In higher rank a similar result holds. This is joint work with Fabien Pazuki.

Daniel Caro (Université de Caen Normandie, France) : Arithmetic $\mathcal{D}$-modules over Laurent series fields

Abstract : Let k be a characteristic p field, V be a complete DVR whose residue field is k and fraction field K is of characteristic 0. We denote by $\mathcal{E}_K$ the Amice ring with coefficients in K, and by $\mathcal{E}_K^\dagger$ the bounded Robba ring with coefficients in K. Berthelot's classical theory of Rigid Cohomology over varieties $X/k((t))$ gives $\mathcal{E}_K$-valued objects. Recently, Lazda and Pál developed a refinement of rigid cohomology,

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i.e. a theory of $\mathcal{E}_K^\dagger$-valued Rigid Cohomology over varieties $X/k((t))$. Using this refinement, they proved a semistable version of the variational Tate conjecture.

In order to get a formalism of Grothendieck's six operations in this context, we need to introduce a theory of arithmetic $\mathcal{D}$-modules with $\mathcal{E}_K^\dagger$-valued cohomology. This is the purpose of this talk. We will explain how to check the differential coherence of the constant coefficient appearing in this context.

Chieh-Yu Chang (National Tsing Hua University, Taiwan) : Periods, logarithms and multiple zeta values.

Abstract : In this talk, we will first recall multiple polylogarithms that are generalization of the classical logarithm. Multiple zeta values are specializations of these special functions and have period interpretation in the picture of mixed Tate motives. We then introduce p-adic multiple zeta values initiated by Furusho in 2004. A classical conjecture asserts that p-adic multiple zeta values satisfy the same linear relations that their corresponding real-valued multiple zeta values satisfy. The main result presented in this talk is to prove a function field analogue of this conjecture (joint work with Y. Mishiba). Key ideas of the proof will be sketched.

Shin Hattori (Tokyo City University, Japan) : Duality of Drinfeld modules and P-adic properties of Drinfeld modular forms.

Abstract : Let p be a rational prime, $q > 1$ a p-power and P a non-constant irreducible polynomial in $\mathbb{F}_q[t]$. The notion of Drinfeld modular form is an analogue over $\mathbb{F}_q(t)$ of that of elliptic modular form. On the other hand, following the analogy with p-adic elliptic modular forms, Vincent defined P-adic Drinfeld modular forms as the P-adic limits of Fourier expansions of Drinfeld modular forms. Numerical computations suggest that Drinfeld modular forms should enjoy deep P-adic structures comparable to the elliptic analogue, while at present their P-adic properties are far less well understood than the p-adic elliptic case.

In this talk, I will explain how basic properties of P-adic Drinfeld modular forms are obtained from the duality theories of Taguchi for Drinfeld modules and finite v-modules.

Satoshi Kondo (HSE, Russia) : Regularity of some quotients of Drinfeld modular schemes, using Dickson's theorem

Abstract : Joint with Seidai Yasuda. Let A be the ring of global sections of a connected projective smooth curve over a finite field minus a closed point, e.g. the polynomial ring $\mathbb{F}_q[t]$ over a finite field $\mathbb{F}_q$. Let d be a positive integer. The moduli M_N^d of Drinfeld modules of rank d with level N structure, where N is a torsion A-module generated by at most d elements is defined. It is an A-scheme, regular and of Krull dimension d. The automorphism group $Aut_A(N)$ of N as A-module acts on the moduli. We give a class of subgroups of $Aut_A(N)$ such that the quotient of M_N^d by the group is regular. Main examples : $N = (A/I)^d$ for some ideal I such that $|V(I)| > 1$; the group $G = GL_d(A/I)$ acts on N and hence on M_N^d. Then, for any maximal parabolic subgroup H of G, the quotient M_N^d/H is regular. The outline of proof follows Katz-Mazur Ch.5 for modular curves. An additional ingredient is Dickson's theorem from modular invariant theory.

Ignazio Longhi (Xian Jiao Tong / Liverpool University, China) : Stark-Heegner points and Shimura reciprocity in the function field setting

Abstract : Darmon proposed the construction (by methods of p-adic analysis) of certain points on elliptic curves defined over $\mathbb{Q}$, which he called Stark-Heegner points (and nowadays are often called Darmon points). He conjectured that these points are algebraic and that the Galois action can be expressed by a suitable version of Shimura reciprocity.

Let F be a global function field and E/F a non-isotrivial elliptic curve over F. It is possible to mimic Darmon's construction of Stark-Heegner points in this setting, replacing classical modular curves with Drinfeld modular curves. Even more, one can prove that in the function field case Stark-Heegner points are indeed algebraic. I will explain this and sketch how one can go on to tackle the Shimura reciprocity (in progress, joint work with M.-H. Nicole).

Yoichi Mieda (University of Tokyo, Japan) : Cohomology of affinoid perfectoid spaces and their reductions.

Abstract : Under some conditions, I will compare the compactly supported l-adic cohomology of an affinoid perfectoid space and that of the reduction of its formal model. I will also discuss my ongoing work on applying it to some cases of Fargues' conjecture on geometrization of the local Langlands correspondence.

Nguyen Quang Do Thong (Université de Besançon, France) : On genus formulas and Greenberg's conjecture.

Abstract : Greenberg's well known conjecture, (GC) for short, asserts that the Iwasawa invariants λ and μ associated to the cyclotomic $\mathbb{Z}_p$-extension of any totally real number field F should vanish. This is a reasonable generalization of the well known Vandiver conjecture concerning the field $\mathbb{Q}(\mu_p)^+$. In his foundational 1976 article, Greenberg has shown two necessary and sufficient conditions for (GC) to hold, in two seemingly opposite cases, when p is undecomposed, resp. totally decomposed in F. In this talk we present an encompassing approach covering both cases and resting only on " genus formulas ", that is (roughly speaking) on formulas which express the order of the Galois (co-)invariants of certain modules along the cyclotomic tower. These modules are akin to class groups, and in the end we obtain several unified criteria, which naturally contain the particular conditions given by Greenberg.

Federico Pellarin (Université de Saint-Etienne, France) : On Drinfeld vector valued modular forms.

Abstract : We will report on certain rigid analytic deformations at the infinite place of Drinfeld modular forms for congruence subgroups of $GL_2(\mathbb{F}_q[T])$ which take values in Tate algebras. These families of Drinfeld modular forms themselves do not exhibit any modular behaviour, but they are, in many cases, entries of vector valued modular forms. We will describe partial structural results and we will point out some open questions.

Lenny Taelman (University of Amsterdam, the Netherlands) : Shtukas and special values of Goss L-functions, after Maxim Mornev

Abstract : I will explain some of the results of Maxim Mornev's thesis. In this thesis, he develops a cohomological theory of shtukas which encapsulates delicate arithmetic information about various function field objects, such as Drinfeld modules or Anderson A-motives. Building on ideas of Anderson, Vincent Lafforgue, and others, he uses his theory to prove new and very general results on values of Goss L-functions. These are characteristic p-valued analogues of the usual "motivic" L-functions, and the results of Maxim Mornev can be seen as analogues of formulas such as the analytic class number formula or the Birch- and Swinnerton-Dyer conjecture.

Floric Tavares-Ribeiro (Université de Caen Normandie, France) : On the Bernoulli-Carlitz numbers and the exceptional zeros of certain L series.

Abstract : Let $k \geq 3$ be an integer. It is conjectured that there exist infinitely many primes p such that the Bernoulli number B_{p-k} is not divisible by p.

L. Carlitz established in the 30's analogies between the rings $\mathbb{Z}$ and $\mathbb{F}_q[\theta]$, where q is the power of a prime. He defined in particular the values of the ζ function at positive integers, as well as the *Bernoulli-Carlitz numbers*, BC_n. We prove for the Bernoulli-Carlitz numbers a result even stronger than what is expected for the classical Bernoulli numbers :

For an integer $k \equiv 1 \mod q-1$, and for all irreducible polynomial $P \in \mathbb{F}_p[\theta]$ with $\deg(P) \gg 0$, $BC_{q^{\deg(P)}-k} \not\equiv 0 \mod P$.

This question is linked with the study of exceptional zeros of certain L series. This is a joint work with B. Anglès and T. Ngo Dac.

Fabien Trihan (Sophia University, Japan) : On the equivariant Tamagawa Number conjecture for abelian varieties over function fields of positive characteristic.

Abstract : We will state a function field analogue of the equivariant Tamagawa number conjecture following the framework of David Burns for semistable abelian over finite everywhere unramified extension of the base field and explain in which cases we can actually prove the conjecture.

Nobuo Tsuzuki (Tohoku University, Japan) : On variation of Newton polygons of F-isocrystals and its application

Abstract : Let $\mathcal{E}$ be a convergent F-isocrystal on a projective smooth curve of characteristic p. We study an estimate of numbers of points at which the Newton polygon of $\mathcal{E}$ jumps. We also discuss variation of Newton polygons in cases of open curves. We apply the estimates to the isotriviality problem of families of curves.

Lucia Di Vizio (CNRS et Université de Versailles-Saint-Quentin, France) :

Abstract :

Chia-Fu Yu (Academia Sinica, Taiwan) : Arithmetic Satake compactifications and Algebraic Drinfeld Modular Forms

Abstract : Drinfeld modular schemes were introduced by Drinfeld in order to prove the Langlands reciprocity of the function field analogue. Compactifying these moduli spaces is one of key steps for realizing the Langlands correpsondence in their l-adic cohomologies. Drinfeld constructed the compactification for rank 2 moduli spaces. Higher rank moduli spaces were constructed by Kapranov, Gekeler and Pink by different methods. In this talk we shall discuss the arithmetic Satake compacfications of Drinfeld moduli schemes of any rank. Applications to algebraic Drinfeld modular forms are addressed. This is joint work with Urs Hartl from Muenster University.

Jing Yu (National Taiwan University, Taiwan) : On Special Values of Arithmetic Drinfeld Modular Forms

Abstract : We are interested in the values of (equal characteristic) arithmetic Drinfeld modular forms at CM points. On the one hand a nice transcendence theory for these special values has been developed. On the other hand, after composing with the absolute value evaluation, these modular forms could correspond to very nice functions on the Bruhat-Tits buildings of the moduli in question. This is then connected with the Langlands program for the global function fields. We shall discuss the progress as well as open questions in the higher rank case.